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EXAMINATION OF INVENTORY MODEL FOR DECAYING ITEMS BELOW PERMISSIBLE DELAY IN PAYMENTS

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ABSTARCT

In India, Operations Research came into existence in 1949 with the opening of an OR unit at the Regional Research Laboratory at Hyderabad. At the same time, another group was set up in the Defence Science Laboratory which devoted itself to the problems of stores, purchases and planning. In 1953, an OR unit was established in the Indian Statistical Institute, Calcutta for the application of OR methods in national planning and survey. In 1955, the OR Society of India was formed which became a member of International Federation of OR Societies in 1959. This paper presents an inventory model for deteriorating items with multivariate demand rate under inflationary environment. Inventory is simply a stock of physical assets having some economic value, which can be either in the form of material, money or labor. Inventory is also known as an idle resource as long as it is not utilized. Inventory may be regarded as those goods, which are produced, stored and used for day today functioning of the organization. This can be in the form of physical resource such as raw materials, semi-finished goods used in the production process, finished products ready for delivery to consumers; human resources such as an utilized labor or financial resource such as working capital etc.

Key words: Inventory, functioning, semi-finished

INTRODUCTION

The first conference of OR Society of India was held in Delhi in 1959. It was felt that the primary necessity of the country is to produce well trained OR practitioners who could tackle the practical problems. In the mean time some of the Institutions started producing OR workers to meet the present and future needs of India. It was also decided to start a Journal of Operations Research which took practical shape in 1963 in the form of 'Opsearch'. The Indian Institute of Industrial Engineers has also promoted the development of Operations Research and its Journals 'Industrial Engineering' and 'Management' publish several papers related to the area. Other Journals of OR are: Journal of National Productivity Council, Material Management Journal of India and the Defence Science Journal. Towards the application of Operation Research in India, Prof. Mahalanobis made the first important application. He formulated the second five year plan with the help of Operation Research techniques to forecast the trends of demand, availability of resources and for scheduling the complex scheme necessary for developing our country's economy. It was estimated that India could become self sufficient on food merely by reducing the wastage of food by 15%. Operation Research techniques are used to achieve this goal. In the indusial sector, in spite of the fact that the opportunities of Operation Research work at present are very much limited, Organics Industries in India are gradually becoming conscious of roll of Operation Research and good number of them have well trained Operation Research teams. With the exception of the ornaments and textile industries, applications of Operation Research in other industries have been more or less equally distributed.

Inventory is any stored resource that is used to satisfy a current or a future need. Raw materials, work in process and finished goods are examples of inventory. Inventory levels for finished goods are direct functions of demand. When determine the demand for completed clothes, dryers; for example, it is possible to use this information to determine how much sheet metal, paint, electric motors, switches and other raw materials and work in process are needed to produce the finished product.

All organizations have some type of inventory planning and control system. A bank has methods to control its inventory of cash. A hospital has methods to control blood supplies and other important items. State and federal governments, schools and virtually every manufacturing and production organization are concerned with inventory planning and control. Studying how organizations control their inventory is equivalent to studying how they achieve their objective by supplying goods and services to their customers. Inventory is the common thread that ties all the functions and departments of the organization together.

Inventories constitute the largest component of current assets in many organizations. Poor management of inventories therefore may results in business failures. A stock-out creates an unpleasant situation for the organization. In case of a manufacturing organization, the inability to supply an item from inventory could bring production process to a halt. Conversely, if a firm carries excessive inventories, the added carrying cost may represent the difference between profit and loss. Efficient inventory control, therefore, can significantly contribute to the overall profit-position of the organization.

REVIEW OF LITERATURE:

Deb & Chaudhuri (1987) have attempted to extend the heuristic of Silver (1973) by allowing shortages which are fully backlogged, but their exposition is incorrect. Dave (1989) has pointed out flaws in the article of Deb & Chaudhuri (1987) and has given the correct analysis. Following the approach of Donaldson (1977), Murdeshwar (1988) tried to derive an exact solution for a finite horizon inventory model to obtain at the optimal number of replenishments, optimal replenishment time, assuming the demand rate to be a linear function of time and considering shortages which are completely accumulated. His exact solution turns out to be incorrect as expression for shortage cost was wrong. Dave (1989) later, has proposed a method for solving the same problem reducing the necessary computations. He has also shown that no shortage period for each cycle is a constant fraction of the corresponding cycle length. Deterioration is one of the important factors to be considered in the study of inventory systems. As defined earlier, deteriorated item does not retain its original value, therefore its effect on inventory system cannot be disregarded.

Ghare & Schrader (1963) were the first to notion of deterioration. They proposed an EOQ model in which items deteriorate at a constant rate. Soon after this, Emmons (1968) followed a similar exposition for the decay of radioactive nuclear generator. This work was extended by Covert & Philip (1973) and Philip (1974) by constructing EOQ models for items with variable rate of deterioration which was further generalized by Shah (1977) allowing shortages and considering general deteriorating function.

Shah & Jaiswal (1977) developed an order level inventory model for constantly deteriorating items where a constant fraction of on hand inventory deteriorate each unit of time. The model is established for both the cases, the deterministic as well as well probabilistic. But they committed a mistake while calculating average inventory holding cost for the system. Authors have actually considered the interval which is not the probabilistic model.

MODEL FORMULATION AND SOLUTION:

The cycle starts with initial lot size Q_0 and ends with zero inventory at time t=T. Then the differential equation governing the transition of the system is given by

$$\frac{dI(t)}{dt} = -Kt - (a + bt + cI(t)), \qquad 0 \le t \le T \qquad \text{...... (1)}$$

With boundary condition I(0) = Q_0

The purchasing cost at different delay periods are

$$C_P = \begin{cases} C_r(1-\delta_1), M = M_1 \\ C_r(1-\delta_2), M = M_2 \\ C_r(1-\delta_3), M = M_3 \\ \infty, M > M_3 \end{cases}$$

Where C_r = maximum retail price per unit.

And M_i (i=1,2,3) = decision point in settling the account to the supplier at which supplier offers δ % discount to the retailer.
Now two cases may occur:

1. When $T \geq M$
2. When $T < M$

Case 1: when $T \geq M$
Solving the equation (1), we get

$$\frac{dI(t)}{dt} + KtI(t) = -(a + bt + cI(t))$$

Using the boundary condition $I(0) = Q_0$, we get

$c = Q_0$

Therefore the solution of equation (1) is

$$I(t) = \left\{ Q_0 - at - \frac{(a+b)}{2} t^2 - \left(\frac{aK}{2} + b \right) \frac{t^3}{3} - \frac{bK}{8} t^4 \right\} e^{-t - Kt^2/2} \qquad 0 \leq t \leq T \qquad \text{.... (2)}$$

In this case it is assumed that that the replenishment cycle T is larger than the credit period M.
The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t) e^{-rt} dt$$

$$\begin{aligned} HC = C_h \Bigg[& \left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \\ & - (1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \\ & - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \Bigg] \qquad \text{..... (3)} \end{aligned}$$

The cost of financing inventory during time span [M,T] is

$$FC = i_c C_P \int_M^T I(t) e^{-r(M+t)} dt$$

$$\begin{aligned} FC = i_c C_P \Bigg[& (1 - rM) \left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \\ & - (1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \\ & - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \\ & - (1 - rM) \left\{ Q_0 M - \frac{a}{2} M^2 - \frac{(a+b)}{6} M^3 - \left(\frac{aK}{2} + b \right) \frac{M^4}{12} - \frac{bK}{40} M^5 \right\}. \\ & + (1+r) \left\{ \frac{Q_0}{2} M^2 - \frac{a}{3} M^3 - \frac{(a+b)}{8} M^4 - \left(\frac{aK}{2} + b \right) \frac{M^5}{15} - \frac{bK}{48} M^6 \right\} \\ & + \frac{K}{2} \left\{ \frac{Q_0}{3} M^3 - \frac{a}{4} M^4 - \frac{(a+b)}{10} M^5 - \left(\frac{aK}{2} + b \right) \frac{M^6}{18} - \frac{bK}{56} M^7 \right\} \Bigg] \qquad \text{..... (4)} \end{aligned}$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p \int_0^M (M-t)(a+bt+cI(t))e^{-rt}dt$$

$$Opp.Cost = i_e p\left[(a+bM)\frac{e^{-rM}}{r^2} - 2b\frac{e^{-rM}}{r^3} + \frac{aM}{r} - \frac{(a-bM)}{r^2} + \frac{2b}{r^3}\right] \quad \text{..... (5)}$$

Therefore, the total cost is given by

TC_{1i}=Purchasing Cost +holding cost +ordering cost +interest charged-interest earned for Mϵ\{M_1,M_2,M_3\}

$$TAC_{1i} = \frac{1}{T}TC_{1i} \quad \text{.... (6)}$$

Case 2 when T < M

In this case, credit period is larger than the replenishment cycle consequently cost of financing inventory is zero. The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t)e^{-rt}dt$$

$$\begin{aligned}HC = C_h\Bigg[&\left\{Q_0T - \left(\frac{a}{2}T^2 + \frac{b}{6}T^3 + \frac{aK}{24}T^4 + \frac{bK}{40}T^5\right)\right\}\\ &-r\left\{\frac{Q_0}{2}T^2 - \left(\frac{a}{3}T^3 + \frac{b}{8}T^4 + \frac{aK}{30}T^5 + \frac{bK}{48}T^6\right)\right\}\\ &-\frac{K}{2}\left\{\frac{Q_0}{3}T^3 - \left(\frac{a}{4}T^4 + \frac{b}{10}T^5 + \frac{aK}{36}T^6 + \frac{bK}{56}T^7\right)\right\}\Bigg]\end{aligned} \quad \text{.....(7)}$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p\left[\int_0^T (T-t)(a+bt)e^{-rt}dt + \int_0^T (M-T)(a+bt)e^{-rt}dt\right]$$

$$= i_e p\left[\int_0^T \left\{aT + (bT-a)t - bt^2\right\}e^{-rt}dt + (M-T)\int_0^T \{a+bt\}e^{-rt}dt\right] \quad \text{..... (8)}$$

Therefore the total cost during the time interval T is given by

TC_{2i}=Purchasing cost +holding cost +ordering cost-interest earned (Opp. cost)

$$TAC_{2i} = \frac{1}{T}TC_{2i} \quad \text{..... (9)}$$

Now, our aim is to determine the optimal value of T and M such that TAC(T,M) is minimized where

$$TAC(T,M) = Inf.\begin{cases}TAC_{1i}(T,M), TAC_{2i}(T,M)\\ where, M\varepsilon(M_1, M_2, M_3)\end{cases} \quad \text{..... (10)}$$

Special case:

Case 1 when there is no deterioration, i.e. K=0, then

$$I(t) = \left\{Q_0 - \left(at + \frac{b}{2}t^2\right)\right\}, \qquad 0 \le t \le T$$

$$\begin{aligned}FC = i_c C_P\Bigg[&\left\{Q_0T - \left(\frac{a}{2}T^2 + \frac{b}{6}T^3\right)\right\} - rM\left\{Q_0 - \left(aT + \frac{b}{2}T^2\right)\right\}\\ &-\frac{K}{2}\left\{\frac{Q_0}{3}T^3 - \left(\frac{a}{4}T^4 + \frac{b}{10}T^5\right)\right\} - r\left\{\frac{Q_0}{2}T^2 - \left(\frac{a}{3}T^3 + \frac{b}{8}T^4\right)\right\}\\ &-\left\{Q_0M - \left(\frac{a}{2}M^2 + \frac{b}{6}M^3\right)\right\} + rM\left\{Q_0 - \left(aM + \frac{b}{2}M^2\right)\right\}\end{aligned}$$

$$+\frac{K}{2}\left\{\frac{Q_0}{3}M^3-\left(\frac{a}{4}M^4+\frac{b}{10}M^5\right)\right\}+r\left\{\frac{Q_0}{2}M^2-\left(\frac{a}{3}M^3+\frac{b}{8}M^4\right)\right\}\Bigg\}\Bigg]$$

$$Opp.Cost=i_e p\left[(a+bM)\frac{e^{-rM}}{r^2}-2b\frac{e^{-rM}}{r^3}+\frac{aM}{r}-\frac{(a-bM)}{r^2}+\frac{2b}{r^3}\right]$$

Case 2: when the demand rate is constant means b=0

$$I(t)=\left\{Q_0-\left(at+\frac{aK}{6}t^3\right)\right\}e^{-Kt^2/2}$$

$$0\le t\le T$$

$$HC=C_h\left[\left\{Q_0T-\left(\frac{a}{2}T^2+\frac{aK}{24}T^4\right)\right\}-r\left\{\frac{Q_0}{2}T^2-\left(\frac{a}{3}T^3+\frac{aK}{30}T^5\right)\right\}\right.$$

$$\left.-\frac{K}{2}\left\{\frac{Q_0}{3}T^3-\left(\frac{a}{4}T^4+\frac{aK}{36}T^6\right)\right\}\right]$$

$$Opp.Cost=i_e p\left[a\frac{e^{-rM}}{r^2}+\frac{aM}{r}-\frac{a}{r^2}\right]$$

NUMERICAL EXAMPLE

We consider the value of the parameters as follows

a=100,		b=50,
C_0=200/Order,		C_h=0.05/unit/month
C_r=125/unit,		p=175/unit,
i_c=0.15,		i_e=0.12,
M_1=3months,		M_2=5months,
M_3=7months,		K=0.001
$_1$=20%	δ	$_2$=15%
$_3$=5%		

The optimal solution are given by

$TC_{11}=5.268\times10^5$	T=32months (for M_1)
$TC_{21}=3.1887\times10^5$	T=27months (for M_2)
$TC_{31}=3.5617\times10^5$	T=8months (for M_3)

CONCLUSION

In this paper the model considered the both, deterioration effect and time discounting. Generally, supplier offer different price discount on purchase of items of retailer at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. Constant deterioration is not a viable concept; hence, we have considered an inventory with deterioration increasing with time. To make our study more suitable to present-day market, we have done our research in an inflationary environment. In totality, the fact that the whole study has been done under the implications of inflation, gives it a viability that makes it more pragmatic and acceptable. The setup that has been chosen boasts of uniqueness in terms of the conditions under which the model has been developed.

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